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Entropy flow in the vicinity of a moving vortex core

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Abstract. Using the electric field determined in our generalized Bardeen–Stephen model, we investigate the entropy flow in the vicinity of a moving vortex core. We show that excited quasiparticles around the vortex core can be driven from the trailing to the leading edge of the core due to the electric force field generated by the vortex motion. From the fact that excited quasiparticles can carry entropy, we determine the rate of the entropy flow and the temperature difference between the trailing and leading edges. These quantities are in qualitative agreement with those of the Clem model.

1. Introduction

A correct account of the energy dissipation in a moving vortex is essential for understanding vortex dynamics. Bardeen and Stephen (BS) [1] and Nozières and Vinen (NV) [2] proposed two of the most widely studied classical models for the dissipative vortex dynamics. However, these models show discordant characteristics, e.g. inconsistent results on the normal core current [3] and the Hall angle, demanding further investigation. In an attempt to resolve the above-mentioned contradictory features of these models, we studied the normal core current of a moving vortex previously [3, 4]. In our earlier microscopic work [4] based on the Bogoliubov–de Gennes formalism and the Galilean unitary transformation, we determined the normal core current and the drag force associated with the uniform vortex motion. In other work based on superfluid hydrodynamics [3], we investigated the BS model which is based on the local London theory. By including the contribution from the nonlinear convective derivative (NCD) in the Euler equation to the force field, we generalized the BS model and obtained a core electric field identical to that of the NV model.

Aside from the BS model and the NV model, another important model for the dissipative vortex motion was proposed by Clem [5]. In order to explain the temperature dependence of the flux flow resistivity, Clem argued that entropy increases at the leading edge of the core where the superconducting region converts to a normal one, and that the reverse occurs at the trailing edge. Thus he proposed that there is irreversible entropy flow from the trailing to the leading edge of the moving vortex core. He also determined the local temperature gradients associated with this transverse [6] entropy flow in the vicinity of the core. The leading edge was found to be cooler than the trailing edge. The above-described temperature gradients give rise to dissipation through irreversible entropy production [7] proportional to the second power of the vortex velocity. This dissipation of the Clem model due to the transverse flow of entropy looks different from that of the BS model

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which is due to the longitudinal (parallel to the transport current) flow of the normal core current. However, it is not clear to what extent these mechanisms of dissipation are additive and to what extent they simply provide alternate ways of looking at the same phenomenon [8].

In this paper, we explain the physical origin of the transverse entropy flow in the Clem model by using our generalized BS model [3]. From the classical hydrodynamic study of the superfluid in our generalized BS model, we obtained a force field different from that of the BS model in the vicinity of a moving vortex core; i.e. in addition to the dipolar electric force field of the BS model, there exists a contribution from the NCD in the Euler equation. Accordingly, excited quasiparticles driven by this force field will show a flow pattern different from that of the BS model. In section 2, we show that the additional force field due to the NCD generates transverse normal currents (flows of excited quasiparticles) from the trailing to the leading edge of the core. Using these transverse normal currents and the fact that excited quasiparticles can carry entropy [9, 10], we determine the rate of transverse entropy flow in section 3. While the entropy flow in the Clem model is due to the temperature gradients (thermal force), it is driven by the electric field associated with the vortex motion in our generalized BS model. Thus we convert our electric transverse force into equivalent thermal gradients in order to compare with the Clem model. The temperature difference between the trailing and the leading edges, which is determined from this effective thermal gradient, is found to be in qualitative agreement with that of the Clem model.

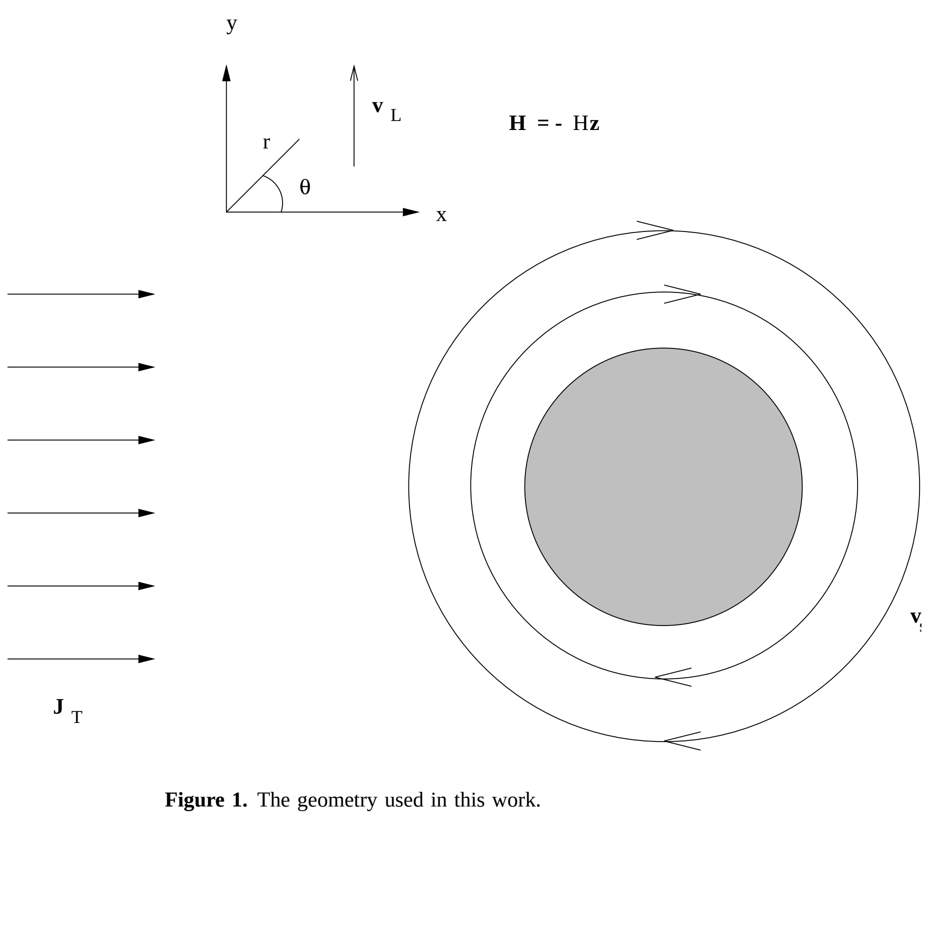

Figure 1. The geometry used in this work.

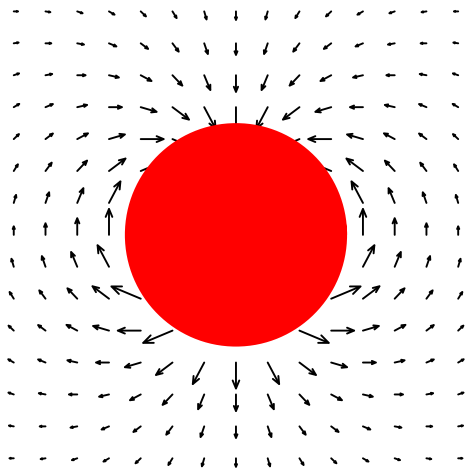

Figure 2. The electric field $(e/m)\boldsymbol{E}_{NCD}$. The length of each arrow is proportional to the strength of the local field.

2. Normal core currents in the generalized BS model

According to our generalized BS model [3], the electric field outside the normal core of a moving vortex can be expressed in terms of the superfluid velocity field as

$$\frac{e}{m}\boldsymbol{E}_1 = -\boldsymbol{v}_L \cdot \boldsymbol{\nabla}\boldsymbol{v}_{s0} + \boldsymbol{\nabla}(\boldsymbol{v}_{s0} \cdot \boldsymbol{v}_{s1}). \tag{1}$$

Here m and e are electron's mass and charge, while $\boldsymbol{v}_L$ and $\boldsymbol{v}_{s0}(r)$ denote the uniform velocity of the vortex and the diamagnetic supercurrent field circulating around the core respectively, as depicted in figure 1. $\boldsymbol{v}_{s1}$ is the leading-order correction to the superfluid velocity field due to the vortex motion and the transport current. We can write $\boldsymbol{v}_{s1}$ as the sum of the velocity fields for the transport current and the backflow [2]:

$$\boldsymbol{v}_{s1} = \boldsymbol{v}_T + \boldsymbol{v}_B. \tag{2}$$

From the definition of the backflow, we notice that v_T is larger than v_B except in the region near the core boundary, $r \simeq a$, where a is the radius of the core. Thus we approximate

$$\boldsymbol{\nabla}(\boldsymbol{v}_{s0} \cdot \boldsymbol{v}_{s1}) \simeq \boldsymbol{\nabla}(\boldsymbol{v}_{s0} \cdot \boldsymbol{v}_T) + \mathrm{O}\left(\frac{a^2}{r^2}\right) \tag{3}$$

and rewrite equation (1) as

$$\frac{e}{m}\boldsymbol{E}_1 \equiv \frac{e}{m}\boldsymbol{E}_{BS} + \frac{e}{m}\boldsymbol{E}_{NCD} \tag{4}$$

where

$$\frac{e}{m}\boldsymbol{E}_{BS} \simeq -\boldsymbol{v}_L \cdot \boldsymbol{\nabla}\boldsymbol{v}_{s0} \qquad \frac{e}{m}\boldsymbol{E}_{NCD} \simeq \boldsymbol{\nabla}(\boldsymbol{v}_{s0} \cdot \boldsymbol{v}_T).$$

Here $(e/m)\boldsymbol{E}_{BS}$ is the dipolar force field of the original BS model [1], and $(e/m)\boldsymbol{E}_{NCD}$ is the additional field originating from the NCD in the Euler equation. The physical origin of $(e/m)\boldsymbol{E}_{NCD}$ is the anisotropy caused by the transport current in the superfluid velocity field of the vortex. The excited quasiparticles existing outside the normal core are driven by this force field in equation (4). Using the London relation, $\boldsymbol{J}_n = n_n(T)e\boldsymbol{v}_n = \sigma_n\boldsymbol{E}_{NCD}$, where $n_n(T)$ is the temperature-dependent density of excited quasiparticles and σ_n is the normal conductivity, we determine the velocity field of the excited quasiparticles outside the core due to $(e/m)\boldsymbol{E}_{NCD}$ as

$$\boldsymbol{v}_n = \frac{e\tau}{m}\boldsymbol{E}_{NCD} \simeq \frac{\hbar\tau v_T}{2mr^2}[-2(\sin\theta)(\cos\theta)\boldsymbol{x} + (\cos^2\theta - \sin^2\theta)\boldsymbol{y}] \tag{5}$$

where τ is the mean free scattering time of excited quasiparticles and θ the angle with respect to $\boldsymbol{x}$. As depicted in figure 2, this velocity field describes the transverse flow of excited quasiparticles flowing from the trailing to the leading edge of the core.

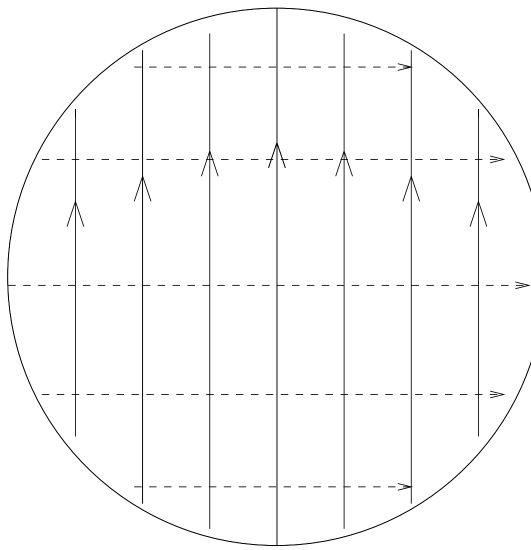

Figure 3. The normal core current due to $e\boldsymbol{E}_c$. Solid (dashed) lines represent the transverse (longitudinal) core current v_{cy} (v_{cx}).

In our earlier work [3], we also determined the electric field inside the normal core:

$$e\boldsymbol{E}_c = \frac{e}{2\pi a^2 c}(2\boldsymbol{v}_T - \boldsymbol{v}_L - \boldsymbol{v}_c) \times \boldsymbol{\phi} \tag{6}$$

where $\boldsymbol{v}_c$ is the velocity of the normal core current. Using this field, we set up a force balance equation for the normal charge carriers in the vortex core. For most of dirty type II superconductors with $\omega_{c2}\tau \ll 1$, where ω_{c2} is the cyclotron frequency at the upper critical field H_{c2}, we determined the normal core current as

$$\boldsymbol{v}_c = v_{cx}\boldsymbol{x} + v_{cy}\boldsymbol{y} \tag{7}$$

where

$$v_{cy} = \tfrac{1}{2}\beta\omega_{c2}\tau(2v_T) + \mathrm{O}((\omega_{c2}\tau)^2)$$
$$v_{cx} = \tfrac{1}{2}\omega_{c2}\tau v_L.$$

Here β is a phenomenological constant associated with the ratio between the effective pinning force f_p and the Lorentz force f_{Lorentz}:

$$\beta = 1 - \frac{2\pi a^2 c}{e}\frac{\gamma' v_L}{2v_T\phi} = 1 - \frac{f_p}{f_{\mathrm{Lorentz}}}. \tag{8}$$

According to equation (7), the normal core current has its transverse velocity component, v_{cy}, as well as its longitudinal component, v_{cx}, identical to those of the BS model. Both components in equation (7) are depicted in figure 3.

3. Transverse entropy flow around the vortex core

While investigating the relation between the flux flow resistivity and the temperature, Clem [5] proposed an entropy flow from the trailing to the leading edge of the moving vortex core. Using the local continuity equation of the entropy, Clem determined the local temperature around the core as

$$T_1(x, y) = -T_0\left[\frac{av_L(S_n - S_s)}{(K_n + K_s)}\right]_{T=T_0} \times \begin{cases} \dfrac{y}{a} & \text{if } x^2 + y^2 < a^2 \\ \dfrac{ay}{x^2 + y^2} & \text{if } x^2 + y^2 > a^2. \end{cases} \tag{9}$$

Here T_0 is the temperature of the core in the absence of vortex motion, S_n (S_s) is the entropy of the normal (superconducting) region and K_n (K_s) is the thermal conductivity of the normal (superconducting) region. From equation (9), we can deduce the transverse thermal gradients parallel to the vortex velocity. This gradients make the transverse entropy flow possible and give rise to the energy dissipation through irreversible entropy production [7]. Using equation (9), we estimate the temperature difference between the leading and the trailing edges as

$$\Delta T \equiv T_1(x, y + a) - T_1(x, y) \simeq T_0\left(\frac{S_n v_L a}{K_n}\right). \tag{10}$$

Here we consider a low-temperature limit near $T \simeq 0$ in which the entropy and the thermal conductivity of the superconducting region are negligible.

In what follows, we will calculate the transverse energy flux using our generalized BS model. The transverse energy flux through the normal core can be written using equation (7) as

$$\frac{\Delta Q_{in,y}}{\Delta t} = 2nv_{cy}\epsilon_0 a \tag{11}$$

where n is the density of normal electrons in the core and ϵ_0 the energy of a normal electron (an excited quasiparticle). If the temperature is not too low, there exists transverse energy flux outside the normal core due to the flow of thermally excited quasiparticles. Thus, using the normal current velocity field for $r > a$ in equation (5), we obtain the energy flux from the trailing to the leading edge of the core as

$$\frac{\Delta Q_{out,y}}{\Delta t} = 2\int_a^\infty \boldsymbol{v}_n(r > a) \cdot \boldsymbol{y}\ \mathrm{d}x = \frac{\hbar\tau v_T}{ma} n_n(T)\epsilon_0. \tag{12}$$

Then the total transverse energy flux is

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta Q_{in,y}}{\Delta t} + \frac{\Delta Q_{out,y}}{\Delta t} = 2(nv_{cy} + n_n v_{n0})\epsilon_0 a \tag{13}$$

where $v_{n0} \equiv \hbar\tau v_T/2ma^2$. This result shows that the transverse energy flux depends on the velocities of the transverse normal core current, v_{cy}, and the external transport current, v_T. Unlike the result in equation (13), the transverse energy flux in the BS model is zero.

In equation (13), the energy flux is driven by the electric field associated with the vortex motion, while it is driven by the temperature gradients (the thermal force) in the Clem model. Thus in order to compare with the result of the Clem model, we convert our electric transverse force into equivalent thermal gradients. In terms of K_n and K_s, we may write the transverse thermal gradients corresponding to equation (13) as

$$\boldsymbol{y} \cdot \boldsymbol{\nabla} T = -\frac{1}{K_n + K_s}\frac{\Delta Q}{\Delta t}. \tag{14}$$

From equation (14), we determine the thermal difference between the leading and the trailing edges at a low-temperature limit near $T \simeq 0$ as

$$\Delta T \simeq \frac{n v_{cy} \epsilon_0 a}{K_n} = T_0 \frac{n v_{cy} \epsilon_0 a}{T_0 K_n} = T_0 \frac{S_n v_{cy} a}{K_n}. \tag{15}$$

Here we define the entropy density of the normal core as $S_n = n\epsilon_0 / T_0$ using the energy and density of normal electrons. The velocity of the transverse core current, v_{cy}, is associated with v_T as shown in equation (7). We may write the following relation between the velocities of the transport current and the vortex:

$$v_T = \alpha v_L \tag{16}$$

because the velocity of the vortex can be approximated as the linear response to the transport current in the flux flow regime. According to the BS model [1], the sample-dependent constant α is given by $\alpha = \eta c / ne\phi$, where η is the viscous drag coefficient and ϕ is the flux quantum. Then we can rewrite equation (15) as

$$\Delta T \simeq (\alpha\beta\omega_{c2}\tau) T_0 \frac{S_n v_L a}{K_n}. \tag{17}$$

The thermal force associated with the thermal difference in equation (17) and the electric force $(e/m)\boldsymbol{E}_{NCD}$ generate equal transverse entropy flux for the moving vortex. Equation (10) of the Clem model and equation (17) of our generalized BS model are qualitatively similar; i.e. both depend on the entropy and the thermal conductivity of the normal core. However, only our expression in equation (17) shows a dependence on the sample-dependent properties such as the mean free scattering time of excited quasiparticles, τ, the relative strength of the pinning force and the Lorentz force, β, and the constant of proportionality between the velocities of the driving transport current and the vortex, α.

4. Conclusions

We studied the entropy flow in the vicinity of a moving vortex core and explained the Clem model within the framework of the BS model. While the BS model describes the longitudinal flow of excited quasiparticles, the Clem model is based on the transverse flow. By using our generalized BS model, we showed that these seemingly inconsistent quasiparticle flows of the two models can be understood in a unified manner as the motion of excited quasiparticles due to the electric force field generated by the vortex motion. From the transverse flow of quasiparticles in our generalized BS model, we determined the transverse entropy flux and the associated temperature difference between the trailing and the leading edges of the core. These quantities from our generalized BS model showed qualitative agreement with those of the Clem model.

Acknowledgments

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